Influence of contact thickness on the acoustic velocities of loose sandstone and its application

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Summary

As the Contact Cement Theory (CCT) proposed by Dvorkin (1994) assumes the contact thickness (the shortest distance between two grains) to be zero in the expressions for the cementation radius, it can't be applied to analyze the influence of contact thickness on the acoustic velocities of loose sandstone and can only predict the acoustic velocities of loose sandstone with porous cementation. To solve this problem, we derive the cementation radius expressions under the assumption that the contact thickness is not zero. Thus, the influence of the contact thickness can be analyzed and the theory can also be extended to predict the acoustic velocities of the loose sandstone with basal cementation. The results show that, the contact thickness can significantly decrease the acoustic velocities. Without considering the influence of contact thickness, the theory will obviously overestimate the acoustic velocities of the loose sandstone with basal cementation.

Introduction

The CCT is proposed by Dvorkin (1994), it analyzes the elastic properties and acoustic velocities of loose sandstone with porous cementation. The model used in the CCT is showed in Figure 1.

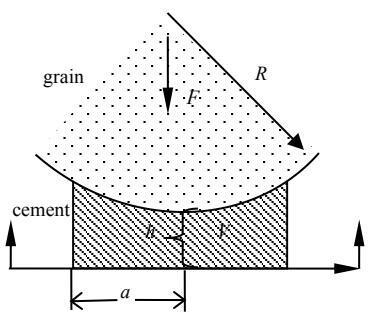


Figure 1: Contact model of CCT. The force performed on the grain is F, the radius of the grain is R, the space between the grains is filled by cement, the cement region is a circle of radius a, the thickness at the center of the cement region (contact thickness) is h, the volume of the cement is V.

Through the mechanics relationship between the grain and the cement, the following expressions for normal stiffness S_n and tangential stiffness S_τ are derived by Dvorkin (1994):

$$S_n = -\frac{4\pi R G_c (1-v_c)}{1-2v_c}\frac{k_n}{\Delta_n} \qquad S_\tau = -2\pi R G_c \frac{k_\tau}{\Delta_\tau}$$

$$k_n = \int_0^\alpha \frac{H_n(t)tdt}{\varepsilon + t^2/2} \qquad k_\tau = \int_0^\alpha \frac{H_\tau(t)tdt}{\varepsilon + t^2/2} \tag{1}$$

Where R is the radius of the grain; G_c is the shear modulus of the cement; v_c is the Poisson's ratio of the cement; k_n and k_τ are proportional to the normal force and tangential force, respectively; $H_n(t)$ and $H_\tau(t)$ are the normal and tangential deformation of the cement, respectively; Δ_n and Δ_τ are the overall normal and tangential deformation, respectively; ε is the ratio of the contact thickness h to the radius of the grain R (normalized contact thickness); α is the ratio of the cementation radius a to the radius of the grain R (normalized cementation radius).

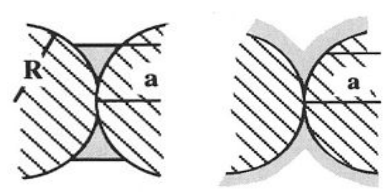
After obtaining the normal and tangential stiffness, the moduli and acoustic velocities of the loose sandstone can be expressed as follows (Digby, 1981; Winkler, 1983):

$$K_{eff} = \frac{n(1-\phi_0)}{12\pi R}S_n \qquad G_{eff} = \frac{n(1-\phi_0)}{20\pi R}(S_n + \frac{3}{2}S_\tau)$$

$$V_p = \sqrt{(K_{eff} + \tfrac{4}{3}G_{eff})/\rho} \qquad V_s = \sqrt{G_{eff}/\rho} \tag{2}$$

Where n is the number of grains around each grain; R is the radius of the grain; Φ_0 is the critical porosity of the loose sandstone; K_{eff} and G_{eff} are the bulk and shear moduli of the loose sandstone; ρ is the density of the loose sandstone; V_p and V_s are the compressional and shear wave velocities of the loose sandstone, respectively.

In order to obtain the moduli and acoustic velocities from (1) and (2), we need the expressions for the normalized cementation radius α. To give the expressions for α, Dvorkin (1996) considered two ways of cementation. They are showed in Figure 2.



(a) (b)

Figure 2: The ways of cementation for loose sandstone, (a) Arrangement 1: cement only accumulates at the grain contacts, (b) Arrangement 2: cement distributes around the grains. Note that the contact thickness is zero.

Under the assumption that the contact thickness is zero, i.e. h or ε=0, Dvorkin gave the following expressions for α:

Arrangement 1: $$\alpha = 2[\frac{\phi_0 - \phi}{3n(1-\phi_0)}]^{0.25} \quad (3\text{-a})$$

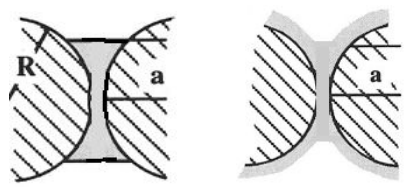
Arrangement 2: $$\alpha = [\frac{2(\phi_0 - \phi)}{3(1-\phi_0)}]^{0.5} \quad (3\text{-b})$$

Where Φ is the porosity of the loose sandstone.

Due to the assumption of zero contact thickness of Eq. (3) , the CCT can only be applied in the loose sandstone with porous cementation (the contact thickness is zero). In order to analyze the influence of contact thickness and extend the theory to the loose sandstone with basal cementation, we derive the new expressions for the cementation radius α under the assumption that the contact thickness is not zero.

The derivation of the general cementation radius expressions

When the contact thickness is not zero, the two ways of cementation can be showed in Figure 3.



(a) (b)

Figure 3: The ways of cementation for loose sandstone, (a) Arrangement 1: cement only accumulates at the grain contacts, (b) Arrangement 2: cement distributes around the grains. Note that the contact thickness is not zero.

Arrangement 1

For Arrangement 1, the cement only accumulates at the grain contacts as Figure 1 and Figure 3(a), through the geometric relations, the relative volume of the cement in the loose sandstone is:

$$C_1 = \frac{(\pi a^2 h + \frac{a^2}{4}\frac{\pi}{R})nm}{\frac{4}{3}\pi\frac{mR^3}{1-\phi_0}} \quad (4)$$

Where n is the number of grains around each grain (coordination number), m is the number of grains in the sandstone.

According to the definition of critical porosity (Nur, 1992), we also have :

$$C_1 = \phi_0 - \phi \quad (5)$$

From (4) and (5), we can get the following equation for α:

$$\frac{3n}{16}\alpha^4 + \frac{3n\varepsilon}{4}\alpha^2 - \frac{\phi_0 - \phi}{1-\phi_0} = 0 \quad (6)$$

By solving Equation (6), the expression for α is :

$$\alpha = \sqrt{-2\varepsilon + 2\sqrt{\varepsilon^2 + \frac{4}{3n}\frac{\phi_0 - \phi}{1-\phi_0}}} \quad (7)$$

If ε=0, Equation (7) can be simplified:

$$\alpha = 2[\frac{\phi_0 - \phi}{3n(1-\phi_0)}]^{0.25} \quad (8)$$

Equation (8) is same with (3-a), which shows that (3-a) is a particular case of (7) for the zero contact thickness and also demonstrates the correctness of (7).

Arrangement 2

For Arrangement 2, the cement is distributed evenly around the grain as Figure 3(b), we can get the following equation for cementation radius from the geometric relation:

$$\frac{3}{2}\alpha^2 + \frac{3}{4}\alpha^4 + \frac{1}{8}\alpha^6 = \frac{\phi_0 - \phi}{1-\phi_0} \quad (9)$$

Because $\alpha<1$, $\alpha^6 \ll \alpha^4 \ll \alpha^2$, α^6 and α^4 can be ignored, then we can get the expression for α with Arrangement 2:

$$\alpha = \sqrt{\frac{2(\phi_0 - \phi)}{3(1-\phi_0)}} \quad (10)$$

(10) is same with (3-b), therefore the expression for α under Arrangement 2 is not related to the contact thickness. However, it should be noted that the elastic properties and acoustic velocities of loose sandstone under this way of cementation is still related to contact thickness due to Equation (1).

The influence of contact thickness on the acoustic velocities of loose sandstone

With new cementation radius expressions (7) and (10), combined with (1) and (2), we can analyze the influence of contact thickness on the acoustic velocities of loose sandstones through numerical calculation. The parameters used are as follows: the grains are quartz, the bulk and shear moduli are 38GPa and 44GPa, respectively, the density of the grain is 2.65g/cm^3; the cement is epoxy, the bulk and shear moduli are 6.8GPa and 2GPa (Dvorkin,1999) , respectively, the density is 0.98g/cm^3; the critical porosity of the loose sandstone equals to that of random dense pack of spherical grains (Dvorkin,1996) and its value is 36%, the corresponding coordination number is 9 (Murphy,1982); the range of the porosity is 15% to 35%; the normalized contact thickness ε changes from 0 to 0.05. The calculation results are showed in Figure 4.

Influence of contact thickness on acoustic velocities

By analyzing Figure 4, we can get the following conclusions: 1) the compressional and shear wave velocities both decrease with contact thickness for both two ways of cementation. 2) The larger the porosity, the larger the influence of the contact thickness on the acoustic velocities. 3) *Vp/Vs* increases with contact thickness for both two ways of cementation. 4) *Vp/Vs* only changes slightly with porosity. For small contact thickness, *Vp/Vs* decreases a little when porosity increases; when the contact thickness becomes larger, the trend goes in the opposite way. 5) When the porosity and the contact thickness is the same, the acoustic velocities under the first way of cementation are obviously larger than those for the second way of cementation.

Prediction of acoustic velocities of loose sandstone with basal cementation

For loose sandstone with basal cementation, the grains don't contact each other, i.e. the contact thickness is not zero. Therefore, we need to consider the influence of contact thickness on the acoustic velocities and the new cementation radius expressions should be used. In order to analyze the effects of contact thickness, we made 18 basally cemented loose sandstone core samples, one of the thin sections of the man-made loose sandstone samples is showed in Figure 5.

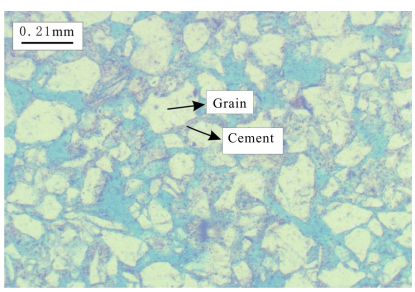


Figure 5: Thin section of the core sample. Note that the grains don't contact each other directly, i.e. the contact thickness is not zero.

In the man-made core samples, the matrix of the samples is composed of the quartz grains, its bulk and shear moduli are 38GPa and 44GPa, the density is 2.65g/cm^3; the cement is mixture of epoxy and kaolinite, its bulk and shear moduli are 2.24GPa and 1.57GPa, the density is 1.38g/cm^3, it is distributed around the grain; according to the volume of the grains (which can be calculated by the whole mass and density of the grain), and the whole volume of the core samples, the critical porosity is 40%, the corresponding coordination number is 8.5(Murphy,1982), the content of cement is the difference value between the critical porosity and the real porosity; the normalized contact thickness ε can be measured through the thin sections and the value is about 0.03.

The acoustic velocities of the dry samples are measured by the Wave Velocity Multi Parameter Meter (WVMPM) made by China University of Petroleum (East China), the frequencies of the compressional and shear wave detector are 0.25MHZ and 0.12MHZ, respectively. The measurement relative uncertainty for the compressional wave velocity is less than 0.5%, and that for the shear wave velocity is less than 1% (Han, 2012). To make sure the accuracy of measurement, the wave velocities are measured 3 times under normal temperature and pressure, the measurement results and the prediction with and without considering the effects of contact thickness are showed in Figure 6. *Vp/Vs* is also showed in Figure 6.

From Figure 6, we can clearly see that, the original CCT without considering the effects of the contact thickness will obviously overestimate the acoustic velocities measured. By considering the effects of the contact thickness, the modified CCT can predict the velocities pretty well. The contact thickness has a significant effect on the acoustic velocities of the loose sandstone with basal cementation , it can obviously decrease the acoustic velocities. It indicates that the contact thickness will make the sandstone with basal cementation much softer than the sandstone with porous cementation (zero contact thickness), thus lead to the low acoustic velocities of the loose sandstone with basal cementation. We also see that *Vp/Vs* changes little with porosity under the constant contact thickness in this case.

Conclusions

New cementation radius expressions for the CCT are derived based on the geometric relations between the grains and cement in the loose sandstone. With the expressions, we can investigate the influence of the contact thickness on the acoustic velocities of the loose sandstone and extend the theory to the prediction of the acoustic velocities of the loose sandstone with basal cementation. The results show that the contact thickness can obviously decrease the acoustic velocities of the loose sandstone. Without considering the influence of the contact thickness, the CCT will obviously overestimate the acoustic velocities of the loose sandstone with basal cementation. It also shows that *Vp/Vs* changes more obviously with contact thickness than porosity.

Acknowledgements

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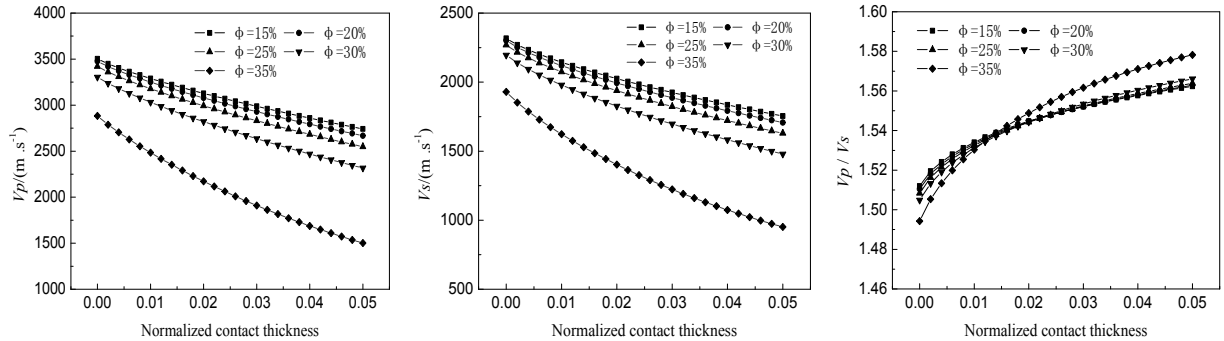
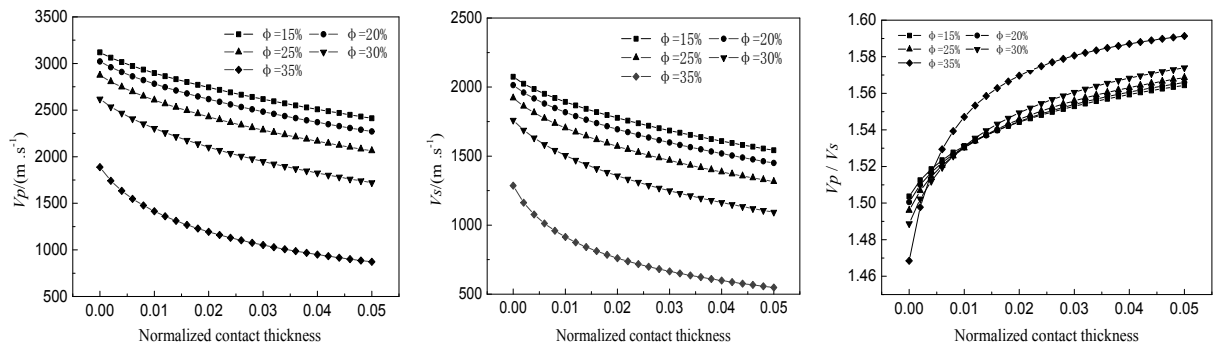
Influence of contact thickness on acoustic velocities

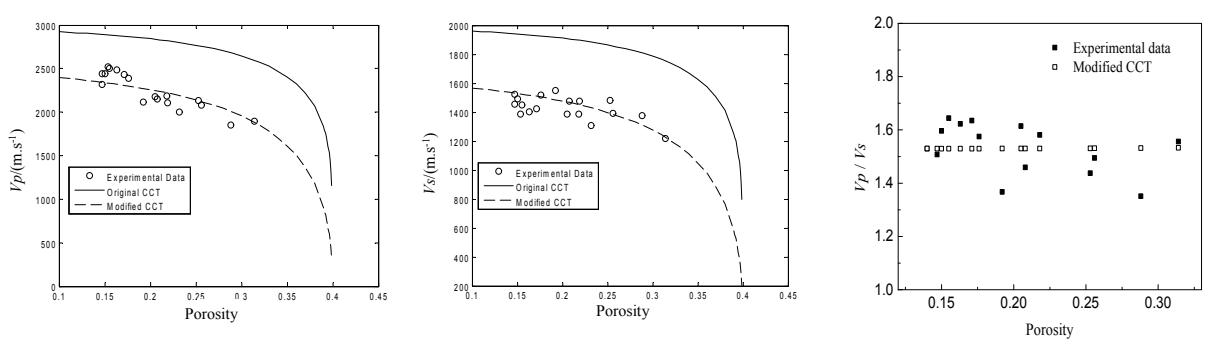


(a) Arrangement 1(cement accumulates at the grain contacts)



(b) Arrangement 2 (cement evenly distributes around the grain)

Figure 4: The influence of the contact thickness on the acoustic velocities of loose sandstone



(a) (b) (c)

Figure 6: Experimental measurement results of acoustic velocities of loose sandstone with basal cementation, the prediction with and without considering the effects of contact thickness and *Vp/Vs* of measured and predicted values. (a) Compressional wave velocity, (b) Shear wave velocity,(c) *Vp/Vs*. Note that the original CCT obviously overestimate the acoustic velocities of the loose sandstone with basal cementation, *Vp/Vs* changes little with porosity under the constant contact thickness.